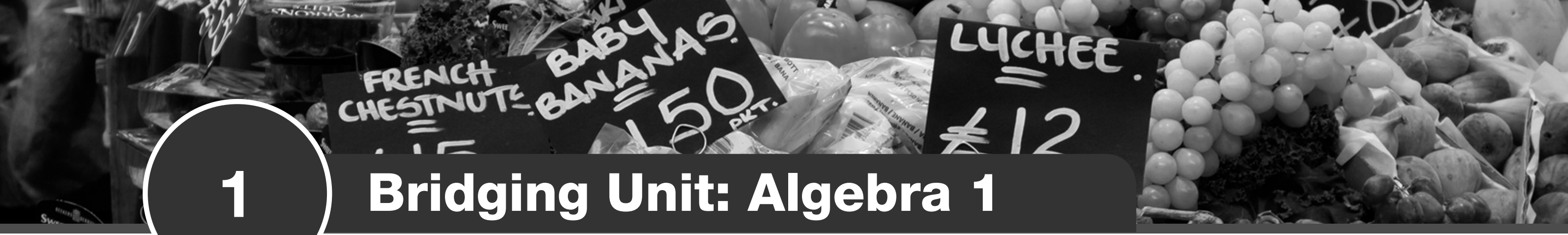

1 Bridging Unit: Algebra 1

Topic C: Factorising quadratics and simple cubics

Bridging to Ch1.4

Expressions such as $5x^2+x$, $2x^2+4$ and x^2+2x-1 are called **quadratics** and can sometimes be factorised into two linear factors. There are three types of quadratics to consider:

1. Quadratics of the form ax^2+bx have a common factor of x so can be factorised using a single bracket and removing the highest common factor of the two terms, e.g. $6x^2+8x=2x(3x+4)$
2. Quadratics of the form x^2+bx+c will sometimes factorise into two sets of brackets. You need to find two constants with a product of c and a sum of b, e.g. $x^2-3x+2=(x-2)(x-1)$ since $-2\times-1=2$ and $-2+-1=-3$
3. Quadratics of the form ax^2-c will factorise if a and c are square numbers. This is called the **difference of two squares**, e.g. $4x^2-9=(2x+9)(2x-9)$

Example 1

Factorise each of these quadratics.

a $9x^2+15x$ **b** $x^2+3x-10$ **c** x^2-16

a $9x^2+15x=3x(3x+5)$

The highest common factor of $9x^2$ and $15x$ is $3x$

b $x^2+3x-10=(x+5)(x-2)$

You need to find two constants with a product of -10 and a sum of 3: $5\times-2=-10$ and $5+-2=3$ so the constants are -2 and 5

c $x^2-16=(x+4)(x-4)$

x^2 and 16 are both square numbers.

Try It 1

Factorise each of these quadratics.

a $14x^2-7x$ **b** x^2-5x+4 **c** x^2-25

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When factorising quadratics of the form ax^2+bx+c with $a\neq 1$, first split the bx term into two terms where the coefficients multiply to give the same value as $a\times c$

Example 2

Factorise each of these quadratics.

a $3x^2+11x+6$ **b** $2x^2-9x+10$

a $3x^2+11x+6=3x^2+9x+2x+6$

$=3x(x+3)+2(x+3)$

$=(3x+2)(x+3)$

b $2x^2-9x+10=2x^2-4x-5x+10$

$=2x(x-2)-5(x-2)$

$=(2x-5)(x-2)$

Split $11x$ into $9x+2x$ since $9\times 2=18$ and $3\times 6=18$

Factorise the first pair of terms and the second pair of terms.

Split $9x$ into $-4x-5x$ since $-4\times -5=20$ and $2\times 10=20$

Factorise the first pair of terms and the second pair of terms.

Try It 2

Factorise each of these quadratics.

a $5x^2+21x+4$ **b** $6x^2+7x-3$ **c** $8x^2-22x+5$

You can use the factors of ax^2+bx+c to find the roots of the **quadratic equation** $ax^2+bx+c=0$

Example 3

Use factorisation to find the roots of these quadratic equations.

a $4x^2+12x=0$ **b** $5x^2=21x-4$

a $4x^2+12x=4x(x+3)$ — Factorise the quadratic.

$4x(x+3)=0 \Rightarrow 4x=0$ or $x+3=0$ — One of the factors must be equal to zero.

If $4x=0$ then $x=0$ and if $x+3=0$ then $x=-3$ — Solve to find the roots.

b $5x^2-21x+4=0$ — Rearrange so you have a quadratic expression equal to zero.

$5x^2-21x+4=5x^2-20x-x+4$ — Write $-21x=-x-20x$ since $-20\times-1=20$ and $5\times4=20$

$=5x(x-4)-(x-4)$

$=(5x-1)(x-4)$ — Factorise the quadratic.

$(5x-1)(x-4)=0 \Rightarrow 5x-1=0$ or $x-4=0$ — The product is zero so one of the factors must be equal to zero.

If $5x-1=0$ then $x=\frac{1}{5}$ and if $x-4=0$ then $x=4$ — Solve to find the roots.

Try It 3

Find the roots of these quadratic equations.

a $6x^2-12x=0$ **b** $4x^2=23x-15$

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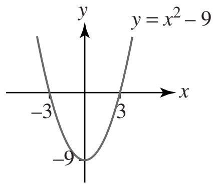
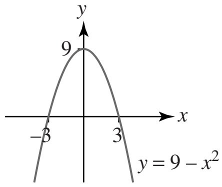
A quadratic function has a **parabola** shaped curve.

When you sketch the graph of a quadratic function you must include the coordinates of the points where the curve crosses the x and y axes.





Example 4

Sketch these quadratic functions.

a $y=x^2+x-6$ **b** $y=-x^2+4x$

a When $x=0$, $y=-6$

Find the y-intercept by letting $x=0$

When $y=0$, $x^2+x-6=0$

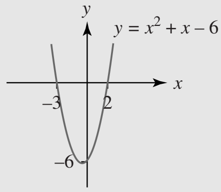
Find the x-intercept by letting $y=0$

$x^2+x-6=(x+3)(x-2)$

Factorise to find the roots.

$(x+3)(x-2)=0 \Rightarrow x=-3$ or $x=2$

Sketch the parabola and label the y-intercept of –6 and the x-intercepts of –3 and 2



b When $x=0$, $y=0$

Find the y-intercept by letting $x=0$

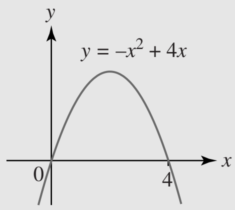
When $y=0$, $-x^2+4x=0$

Find the x-intercept by letting $y=0$

$-x^2+4x=-x(x-4)$

$-x(x-4)=0 \Rightarrow x=0$ or $x=4$

Factorise to find the roots.



Sketch the parabola, it will be this way up since the x^2 term in the quadratic is negative. Label the x and y intercepts.

Try It 4

Sketch these quadratic functions.

a $y=x^2-25$

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Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

b $y = x^2 + 10x + 25$

c $y = 5x - x^2$

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Bridging Exercise Topic C

Bridging to Ch1.4

1 Fully factorise each of these quadratics.

a $3x^2+5x$

b $8x^2-4x$

c $17x^2+34x$

d $18x^2-24x$

2 Factorise each of these quadratics.

a x^2+5x+6

b $x^2-7x+10$

c x^2-5x-6

d $x^2+3x-28$

e x^2-x-72

f $x^2+2x-48$

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g $x^2-12x+11$

h $x^2-5x-24$

3 Factorise each of these quadratics.

a x^2-100

b x^2-81

c $4x^2-9$

d $64-9x^2$

4 Factorise each of these quadratics.

a $3x^2+7x+2$

b $6x^2+17x+12$

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c $4x^2-13x+3$

d $2x^2-7x-15$

e $2x^2+3x-5$

f $7x^2+25x-12$

g $8x^2-22x+15$

h $12x^2+17x-5$

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5 Fully factorise each of these quadratics.

a $16x^2 - 25$

b $4x^2 - 16x$

c $x^2 + 13x + 12$

d $3x^2 + 16x - 35$

e $x^2 + x - 12$

f $100 - 9x^2$

g $2x^2 - 14x$

h $20x^2 - 3x - 2$

 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

6 Use factorisation to find the roots of these quadratic equations.

a $21x^2 - 7x = 0$

b $x^2 - 36 = 0$

c $17x^2 + 34x = 0$

d $6x^2 + 13x + 5 = 0$

e $4x^2 - 49 = 0$

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f $x^2 = 7x + 18$

g $x^2 - 7x + 6 = 0$

h $21x^2 = 2 - x$

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i $17x = 5x^2 + 6$

j $16x^2 + 24x + 9 = 0$

k $9x^2 + 4 = 12x$

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l $40x^2+x=6$

7 Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a $y=x(x-3)$

b $y=-x(3x+2)$

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c $y = x(3 - x)$

d $y = (x + 2)(x - 2)$

e $y = (x + 4)^2$

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f $y = -(2x+5)^2$

g $y = (x-5)(x+2)$

h $y = (x+1)(5-x)$

 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

8 Sketch each of these quadratic functions, labelling where they cross the *x* and *y* axes.

a $y = x^2 + 6x$

b $y = 3x^2 - 12x$

c $y = x^2 - 121$

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d $y = x^2 - 3x - 10$

e $y = -x^2 + 3x$

f $y = 15x - 10x^2$

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g $y = 49 - x^2$

h $y = -x^2 + 2x + 3$

i $y = x^2 - 4x + 4$

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j $y = -x^2 + 14x - 49$

k $y = 3x^2 + 4x + 1$

l $y = -2x^2 + 11x - 12$

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